PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Relation between Magnetic Stress and Magnetic Deformation in Nickel

E. Taylor Jones

Phil. Trans. R. Soc. Lond. A 1897 **189**, 189-200
doi: 10.1098/rsta.1897.0008

VIII. *On the Relation between Magnetic Stress and Magnetic Deformation in Nickel.*

By E. Taylor Jones, *D.Sc.*

Communicated by Professor Andrew Gray, *F.R.S.*

Received January 13,—Read February 25, 1897.

The object of the experiments described below was to determine how much of the contraction, which occurs in a nickel wire when magnetised, is due to stresses brought into play by magnetisation. This has been attempted by several experimenters, but it has been assumed by some that the magnetic stress concerned is a contracting stress of magnitude $B^2/8\pi$, an assumption which has only recently been shown to be unjustifiable.*

That the system of stresses in a magnetic field, described by Maxwell ("Electricity and Magnetism," Art. 642) as explaining the observed forces between magnetic bodies, is not sufficient to account for the observed deformation of bodies in the field, is clear from the fact that these stresses increase continually with the magnetisation and field-strength, while the deformation in iron and cobalt becomes reversed when the field reaches a certain value. It can also be shown that Maxwell's stresses would produce no change in the dimensions of a ring magnetised by a uniform circumferential field, whereas Bidwell† has found considerable change both in the diameter and the volume of rings of iron.

The theory of magnetic stress has, however, been extended, chiefly by v. Helmholtz,‡ Kirchhoff,§ Professor J. J. Thomson,‖ and Hertz,¶ who have shown that, in addition to Maxwell's distribution of stress, there are other stresses in a magnetised body due to the fact that the magnetisation depends upon the strain in the body.

Hitherto no direct experiments have been made with the object of ascertaining whether these stresses, in addition to those of Maxwell, are sufficient to account for the observed deformation of bodies placed in a magnetic field. The facts that "Villari reversals" of opposite kinds exist in iron and cobalt, and also opposite reversals of magnetic deformation, and that in nickel (at any rate at moderate field-

* C. Chree, 'Nature,' January 23, 1896; H. Nagaoka and E. T. Jones, 'Phil. Mag.,' May, 1896.

† S. Bidwell, 'Proc. Roy. Soc.,' vol. 55, p. 228; vol. 56, p. 94.

‡ 'Wied. Ann.,' vol. 13, p. 400, 1881.

§ 'Wied. Ann.,' vol. 24, p. 52, 1885.

‖ 'Applications of Dynamics to Physics and Chemistry,' p. 48, 1888.

¶ 'Ausbreitung der Elektrischen Kraft,' p. 275, 1892.

10.6.97

strengths) there is neither Villari reversal nor reversal of deformation, suggest that there is a close connexion between the two phenomena, and this general similarity between the effects of stress on magnetisation, on the one hand, and magnetic deformation on the other, has led some experimenters to assume that magnetic deformation is completely accounted for by the above stresses, and to calculate from the observed deformation the general effect of stress on magnetisation.*

Before this assumption can be legitimately made, however, it is necessary to make direct experiments to test its truth, and it was with this object that the following experiments were instituted.

The experiments were made in the Physical Laboratory of the University College of North Wales, and, before proceeding to describe them, I wish here to express my great indebtedness to Professor A. GRAY for kindly placing at my disposal all the necessary apparatus, and for many valuable suggestions.

Theory.

One method of experimenting, suggested by Mr. NAGAOKA and the present writer,† would be to measure both the effect of hydrostatic pressure on magnetisation and the magnetic deformation of a ring of soft magnetic material. From these measurements a direct comparison could be made of the observed deformation and the value calculated from the theory of KIRCHHOFF.

There is however another method which, though theoretically not so simple, is probably easier to carry out experimentally. It has been shown by CANTONE‡ that the elongation δl of an ellipsoid of revolution of great eccentricity and soft magnetic material, when placed in a uniform longitudinal field H is, on KIRCHHOFF's theory, given by

$$\frac{\delta l}{l} = \frac{4\pi I^2}{3E}\left(\frac{1+\theta}{1+2\theta}\right) + \frac{IH}{2E(1+2\theta)} - \frac{\kappa' H^2}{2E(1+2\theta)} - \frac{\kappa'' H^2}{2E} \quad . \quad . \quad . \quad (1),$$

where

l = length of ellipsoid,

I = magnetisation,

κ', κ'' are the coefficients of change of magnetic susceptibility with change of density and of elongation§ respectively,

E = YOUNG's Modulus for the material, and θ is defined by

$$\frac{E}{2}\frac{1+2\theta}{1+3\theta} = \text{rigidity} = n.$$

* CANTONE, 'Mem. R. Acc. Linc.,' ser. 4, vol. 6, 1890; WINKELMANN, 'Handbuch der Physik,' Bd. 3, Part II., p. 250, 1895.

† *Loc. cit.*, p. 461.

‡ *Loc. cit.*

§ The fundamental equation defining κ' and κ'' is $I = \{\kappa - \kappa'(e+f+g) - \kappa'' e\}$ H, where e, f, g are the dilatations parallel to the direction of magnetisation, and to two axes at right angles to it.

It can also be shown that if a tension δP per unit area be applied to a long uniform wire of the material and produce a small increase δI in the magnetisation, then

$$\frac{E}{H}\frac{\delta I}{\delta P} = -\frac{\kappa'}{1+2\theta} - \kappa'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Between (1) and (2) the quantity $\left(\frac{\kappa'}{1+2\theta} + \kappa''\right)$ can be eliminated, and the resulting equation is

$$\frac{\delta l}{l} = \frac{4\pi I^2}{3E} \cdot \frac{1+\theta}{1+2\theta} + \frac{IH}{2E(1+2\theta)} + \tfrac{1}{2}H\frac{\delta I}{\delta P} \quad . \quad . \quad . \quad . \quad . \quad (3).$$

In order to compare this result with experiment a long cylindrical specimen should first be used to determine I and $\delta I/\delta P$ for several constant field-strengths. Then the specimen should be turned down to the form of an ellipsoid of revolution and the magnetic elongation α* measured with the specimen in the same magnetic state as that in which I and $\delta I/\delta P$ were determined.

At first it was intended to do this, but afterwards it was thought sufficient to use a wire of the material (of length very great in comparison with its thickness) in both parts of the experiment, thus assuming that the elongation of a long thin wire is the same as that of an ellipsoid of revolution having the same dimensional ratio.† The general agreement between the results of different experimenters, who in measuring magnetic elongation have used wires, strips and ellipsoids of various shapes and degrees of purity, shows that this is at least approximately the case.

It can also be proved independently that in a uniformly magnetised wire the term $\frac{1}{2}H\frac{\delta I}{\delta P}$, which is generally by far the greatest on the right-hand side of (3), represents the elongation due to those stresses which arise in consequence of the fact that magnetisation depends upon strain. For the potential energy, which unit volume has in consequence of the magnetisation, is $-\frac{1}{2}HI$.‡ Hence, if the body has dilatations e, f, g, parallel to the axes of x, y, z, respectively, the part of the Lagrangian Function depending on magnetisation and strain coordinates is (supposing the quantities to vary infinitely slowly)

$$L = \tfrac{1}{2}HI - \tfrac{1}{2}m(e+f+g)^2 - \tfrac{1}{2}n(e^2+f^2+g^2-2ef-2fg-2ge).\S$$

* The symbol α is here used for the observed elongation of the specimen, *i.e.* the ratio of the observed increase of length to the whole length.

† A change in the state of the surface of the specimen would be caused by the turning down, which would make that process inadvisable.

‡ MAXWELL, "Electricity and Magnetism," vol. 2, § 632.

§ J. J. THOMSON, *loc. cit.*

Professor J. J. THOMSON has shown that the strains due to the dependence of magnetisation upon strain are given by the equations

$$\partial L/\partial e = 0, \qquad \partial L/\partial f = 0, \qquad \partial L/\partial g = 0.$$

i.e.,

$$\tfrac{1}{2}H\frac{\partial I}{\partial e} - m(e+f+g) - n(e-f-g) = 0,$$

$$\tfrac{1}{2}H\frac{\partial I}{\partial f} - m(e+f+g) - n(f-g-e) = 0,$$

$$\tfrac{1}{2}H\frac{\partial I}{\partial g} - m(e+f+g) - n(g-e-f) = 0.$$

In the case of a long cylindrical wire with axis along ox, we have $g=f$, and $\partial I/\partial g = \partial I/\partial f$, hence solving for e

$$ne = \tfrac{1}{2}\frac{m}{3m-n}H\frac{\partial I}{\partial e} - \tfrac{1}{2}\frac{m-n}{3m-n}H\frac{\partial I}{\partial f} = \tfrac{1}{2}\frac{m}{3m-n}H\left(\frac{\partial I}{\partial e} - \frac{m-n}{m}\frac{\partial I}{\partial f}\right). \quad . \quad (4).$$

Now if a tension δP per unit area be applied to the wire and produce an increase δI in the magnetisation and strains δe, $\delta f = \delta g$, then

$$\delta I = \frac{\partial I}{\partial e}\delta e + 2\frac{\partial I}{\partial f}\delta f.$$

But if the elastic properties of the material are not altered by magnetisation,

$$\delta f = -\tfrac{1}{2}\frac{m-n}{m}.\delta e,$$

therefore

$$\delta I = \left(\frac{\partial I}{\partial e} - \frac{m-n}{m}\frac{\partial I}{\partial f}\right)\delta e.$$

Hence (4) becomes

$$ne = \tfrac{1}{2}\frac{m}{3m-n}H\frac{\delta I}{\delta e},$$

therefore

$$e = \frac{1}{2E}H\frac{\delta I}{\delta e} = \tfrac{1}{2}H\frac{\delta I}{\delta P}.$$

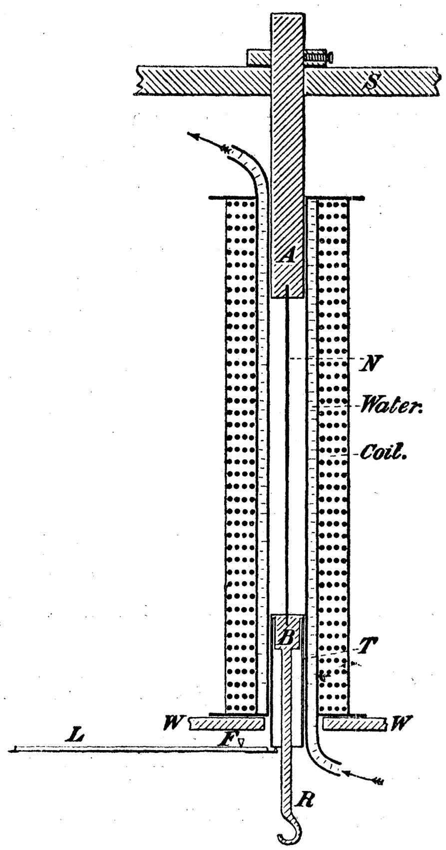
This strain will be the same at all points of the wire, which is supposed uniformly magnetised, and will therefore be that part of α which is due to these stresses.

Apparatus.

In the experiments nickel was used because the quantities to be measured are both greater and of a simpler nature than in the other magnetic metals. The specimen was an annealed wire of length 85·2 centims. and mean diameter 1·65 millims., containing about 98 per cent. of nickel and traces of iron and cobalt. This was obtained from Messrs. JOHNSON, MATTHEY and Co.

The magnetising coil was over a metre long, and consisted of seven layers of No. 18 copper wire wound on a hollow brass core provided with a water-jacket through which water could be made to flow steadily. The coil was mounted on a stand W (fig. 1) in a vertical position.

Fig. 1.



The ends of the nickel wire N were soldered into two brass pieces, the upper of which, A, was a long cylindrical bar, and the other, B, a short block of brass prolonged downwards by a tube T* closed at the lower end. Both bar and tube fitted loosely in the core of the coil, the bar projecting and being fixed to a rigid

* It was at first intended to measure YOUNG's Modulus for the nickel wire, a measurement for which the above apparatus was not very suitable, and which was moreover afterwards rendered unnecessary.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

support S above the coil, and the tube projecting slightly from the coil below. A thin brass rod R was screwed into the block B, and passing downwards through an opening in the lower end of the tube T, was attached to a scalepan below.

The method adopted for observing changes in the length of the nickel wire was essentially the same as has been used by several experimenters. At one end of a long light lever L, of wood, a sharp gun-metal point pressed upwards against the lower end of the tube T, and the fulcrum F consisted of two gun-metal points resting on two plates of glass fixed on a support which was rigidly connected with the upper support S; the connexion is not shown in the diagram (fig. 1). At the end of the other (longer) arm of the lever, a small metal plate was attached, at one point of which a slight depression was made; in this depression rested one of the three needle-point feet of a small light table on which a plane mirror was mounted. The two other feet rested on a fixed horizontal metallic support. Deflections of this mirror were observed by a telescope and vertical scale at a distance of about 3 metres.

Levers of various lengths were tried; the arms of the lever ultimately chosen were 3·275 centims. and 60·92 centims. long, measured from the line joining the fulcrum points to the point at one end, and the depression at the other, respectively. The distance of the moving foot of the mirror table from the line joining the two fixed feet was 8·97 millims. The magnifying power of this arrangement was 12,300.

The nickel wire was thus suspended inside but independently of the coil. The effect of any "suction" between the coil and the wire was, however, quite negligible, on account of the great rigidity of the support S, and because this support was rigidly connected with the fulcrum support of the lever, so that any displacement of the upper support involved a similar displacement of the fulcrum support, and therefore no *turning* of the lever. This was tested by observing the scale deflections caused by placing weights on the upper support S. Even if the wire had only one pole, the error due to this cause would be less than 0·1 per cent. of the deflection observed when the current was made.

The error due to the slight non-uniformity of the field of the coil towards the ends of the nickel wire was also of the same order.

The magnetising current was generally measured by a Kelvin graded galvanometer, standardised by electrolysis of copper, the smaller currents by a Kelvin centiampere balance.

For measuring the magnetisation, which was done in separate experiments, 600 turns of No. 40 double-silk covered and shellacked copper wire were wound in one layer near the middle of the nickel wire. These were connected to a ballistic galvanometer which was wound with wire calculated to give the greatest sensitiveness.* A long solenoid with secondary coil was used to standardize this galvanometer in the usual way.

* See A. GRAY, "Absolute measurements in Electricity and Magnetism," vol. 2, Part II., p. 369.

Measurement of I *and* δI/δP.

The magnetisation was measured by observing the deflection of the galvanometer needle when the magnetising current was reversed. The sectional areas of the nickel wire and of the coil wound on it being known, and the galvanometer being frequently standardised, the induction in the wire, and hence the magnetisation, were calculated.

In order that equation (3) should hold, it was necessary that the nickel wire should be in the same magnetic state in the magnetisation and in the elongation experiments. Hence, both experiments were made with increasing reversals of magnetisation. Also, since during the reversals of magnetisation the magnetic stress is alternately applied and removed, the weight used in measuring δI/δP was added and removed several times before readings were taken. An initial weight of 1 kilogram was always kept in the scalepan and the effect observed at several fields of adding a few kilograms, the magnetisation being measured before and after the additional load was applied.

An increase of tension always caused a diminution of magnetisation, which was not in general proportional to the weight added.* [Added May 15.—This depends on the field-strength. At low fields, within the range of tensions used in the present experiments, the effect of tension in reducing magnetisation diminishes slightly as the tension is increased, at stronger fields the opposite is th ecase; in other words, at low fields $\partial^2 I/\partial P^2$ is positive, at high fields negative. This may partly explain the fact observed by BIDWELL that at low fields increase of tension diminishes the magnetic contraction in nickel, at high fields increases it. For if part of the contraction is represented by

$$-e = \tfrac{1}{2} H \frac{\partial I}{P\partial},$$

then

$$\frac{\partial e}{\partial P} = -\tfrac{1}{2} H \frac{\partial^2 I}{\partial P^2},$$

which is negative at low, positive at high fields.] It could, however, be assumed that the change of magnetisation, divided by the increase of tension per unit area of section of the wire, gave the value of δI/δP for the mean load. The curve of increasing reversals for the mean load was then determined.

Several loads were tried, but the effects of adding 2 and 7 kilograms respectively sufficiently show the nature of the results. Thus, the curves of increasing reversals of magnetisation, and the corresponding values of δI/δP, were determined with 2 and 4·5 kilograms in the scalepan, the total tensions being 2·4 and 4·9 kilograms.

Since the dimensional ratio of the nickel wire was about 500, the mean demagnetising force† was ·00018 I. This was never more than about 0·1 per cent. of the magnetising force due to the coil.

* See EWING, 'Magnetic Induction in Iron and other Metals,' p. 196, 1893.

† DU BOIS, 'The Magnetic Circuit in Theory and Practice,' p. 41, 1896.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

All ballistic measurements were repeated a number of times, and mean values taken, to eliminate, as far as possible, small errors of observation.

Measurement of Magnetic Change of Length.

It was necessary to determine the change of length of the nickel wire under tensions of 2·4 and 4·9 kilograms, for a series of increasing fields, the current being reversed a few times at each step.

Instead of doing this directly, by first demagnetising the wire and then applying the current, it was found more convenient to measure the temporary and residual changes of length separately.

The temporary change of length was determined by first making and reversing several times a measured current, and then observing the scale deflection caused by breaking the current. The reversed current was then made, the deflection observed on breaking it, and the mean of the two deflections taken. The wire always lengthened when the current was broken.

No complete hysteresis loops were obtained, but the opportunity was taken of noting how much of the residual contraction could be removed by applying a reversed field. Thus each time the current was broken a small reversed current was applied, causing an increase of length, and this current was gradually increased until the length reached a maximum, and at higher fields again diminished.

Next a series of residual contractions was determined, the wire being demagnetised by reversals before each reading and a current made for a short time, the scale reading being taken before the current was made and after it was broken.

The residual contraction was always found to be greater than the contraction which can be removed by applying an increasing reversed current. This has also been observed by Mr. NAGAOKA.*

As the magnetising field is increased both the residual and the "removable" contraction at first increase and then become nearly constant, the former being then about $2{\cdot}5 \times 10^{-6}$ and the latter $1{\cdot}9 \times 10^{6}$ of the length of the wire.

The reversed field corresponding to minimum contraction was about 16 C.G.S.

The temporary contraction was, however, still increasing in the highest fields employed, its value at $H = 350$ being about 33×10^{-6}, with a tension of 4·9 kilograms.

The curve representing the total contraction was obtained by adding the ordinates of the "temporary" and "residual" curves.†

The difference between the tensions 2·4 and 4·9 kilogs. weight was not sufficient

* 'Phil. Mag.,' Jan., 1894, p. 131.

† Since about a centimetre of the nickel wire at each end was soldered in the brass pieces A and B, the length of wire whose changes were measured was only 83·2 centims. This exposed length l was used in calculating α.

to cause much difference in the curves. At the highest fields used the contractions were, however, rather greater with the greater tension.

At low and moderate field-strengths the contraction is less, but at strong fields greater, than the value obtained by Mr. NAGAOKA for an ellipsoid of nickel.*

Water was kept flowing through the coil for a considerable time before readings were taken, and all temperature changes took place so slowly that their effects were easily distinguishable from the magnetic effect.

All the readings were repeated several times, and the points representing the results always lay very near the curve (fig. 3).

Final Results.

On the right-hand side of the equation (3) the third term is much greater than the other two. Hence it was not necessary to know E and θ with very great accuracy. The value of YOUNG's Modulus for a 98·1 per cent. nickel wire has recently been given by MEYER† as $21 \cdot 3 \times 10^{11}$ C.G.S. This value of E was used, and the rigidity was measured by observing the torsional oscillations of the wire, to which a cylindrical vibrator was attached. This gave $n = 7 \cdot 75 \times 10^{11}$ approximately. Hence $\theta = 1 \cdot 5$.

The greatest numerical value of $\frac{1}{2} H (\delta I/\delta P)$ obtained was about $8 \cdot 2 \times 10^{-6}$, while the greatest value of the first two terms in (3), representing the elongation due to MAXWELL's stresses, was but $0 \cdot 29 \times 10^{-6}$.

Tables I and II contain the values of H, I, the change δI of magnetisation caused by increasing the tension by δP per unit area of section, the values of $\delta l/l$ calculated from equation (3), and the observed total contraction $-\alpha$.

TABLE I.

Load = 2·4 kilogs. $\delta P \times$ section of wire = 2 kilogs. Temperature = 6° C.

H.	I.	δI.	$\frac{\delta l}{l} \cdot 10^6$ calculated.	$\alpha \cdot 10^6$ observed.	$\left(\alpha - \frac{\delta l}{l}\right) 10^6$.	$I^4 \cdot 10^{-6}$.
7·49	31·0	− 1·6	− 0·063	− 0·1	− ·037	0·92
12·36	105	− 6·4	− 0·412	− 0·3	+ ·11	121·5
36·3	270	− 17·0	− 3·23	− 5·2	− 1·97	5316
64·3	332	− 15·3	− 5·15	− 11·35	− 6·2	12,140
85·5	365	− 13·4	− 6·00	− 15·05	− 9·05	17,750
107·5	389	− 13·0	− 7·30	− 18·35	− 11·05	22,890
159·0	428	− 9·7	− 8·07	− 23·6	− 15·53	33,550
212·0	452	− 5·0	− 5·45	− 27·4	− 21·95	41,730

* 'Wied. Ann.,' vol. 53, p. 487, 1894.

† 'Wied. Ann.,' vol. 59, p. 668, 1896.

TABLE II.

Load = 4·9 kilogs. δP × section of wire = 7 kilogs. Temperature = 6° C.

H.	I.	δI.	$\frac{\delta l}{l} . 10^6$ calculated.	$\alpha . 10^6$ observed.	$\left(\alpha - \frac{\delta l}{l}\right) 10^6$.	$I^4 . 10^{-6}$.
36·8	243	− 63·0	− 3·5	− 5·3	− 1·8	3490
65·2	308	− 60·0	− 5·9	− 11·5	− 5·6	9000
88·1	343	− 54·0	− 7·17	− 15·45	− 8·3	13,850
129·0	386	− 40·3	− 7·81	− 20·75	− 12·9	22,200
164·8	415	− 32·0	− 7·89	− 24·2	− 16·3	29,650
213·0	443	− 22·4	− 7·09	− 28·0	− 20·9	38,500
243·1	456	− 18·0	− 6·46	− 30·0	− 23·5	43,250
291·5	471	− 12·1	− 5·14	− 32·6	− 27·5	49,200
345·0	480	− 11·1	− 5·59	− 34·8	− 29·2	53,100

Fig. 2.

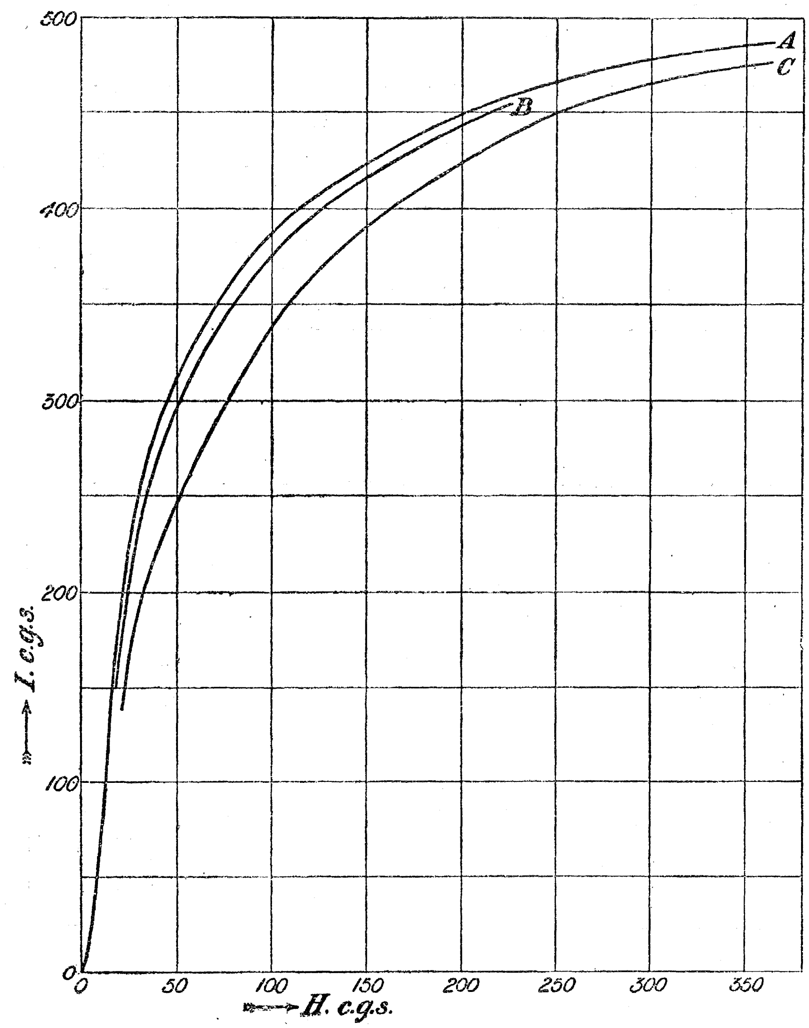


Fig. 3.

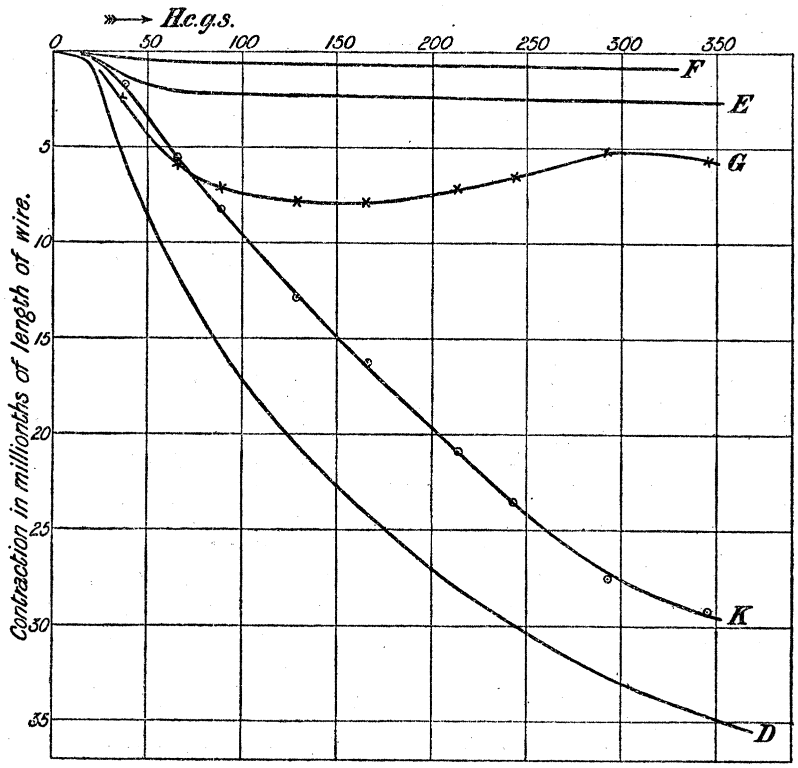
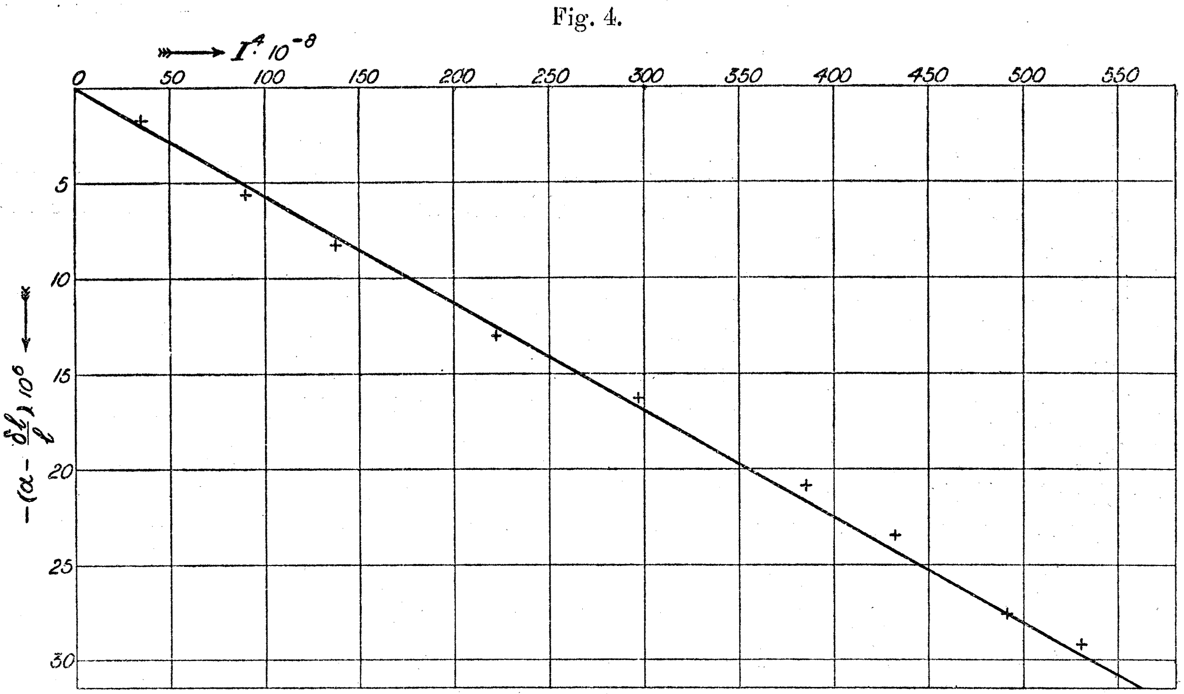


Fig. 4.



In fig. 2, Curve A is the curve of increasing reversals of magnetisation with a load of 1·4 kilogs. The difference of the ordinates of Curves A and B represents the diminution of magnetisation caused by adding two kilogs. to the load, and the difference of ordinates of A and C represents the effect of adding seven kilogs. to the load.

In fig. 3 curve D represents the observed total contraction $-\alpha$, curve E the residual contraction after the field H has been removed, F the least contraction attainable by reversing the current after the field H has been removed, G the value of $\delta l/l$ calculated from equation (3), and K the difference $\alpha - \delta l/l$, all as functions of the field H, the load being 4·9 kilogs. The curve K therefore represents the contraction in nickel corrected for the effects of known stresses.

In the last columns of Tables I and II, the corresponding values of I^4 are given. It will be seen that these numbers are approximately proportional to the numbers in the preceding column representing the values of $\alpha - \delta l/l$.

This is also shown in fig. 4, where the abscissæ of the points . . . + + + are proportional to I^4, and the ordinates to the corrected contraction $-(\alpha - \delta l/l)$ for the load 4·9 kilog. The points all lie, to within about 5 per cent., on a certain straight line passing through the origin. An error of 5 per cent. in the values of I^4 and $\delta I/\delta P$ might be caused by much smaller errors in the measurement of I. Especially was this the case with the other load 2·4 kilog., for the change of magnetisation δI in this case (Table I) was smaller, and therefore more difficult to measure accurately, though even in this case there is no regular deviation from proportionality between I^4 and $\alpha - \delta l/l$.

With the load 4·9 kilog., however, the change of length α observed in the nickel wire is closely represented by the equation

$$\alpha = cI^4 + \frac{4\pi}{3E} \cdot \frac{1+\theta}{1+2\theta} \cdot I^2 + \frac{HI}{2E(1+2\theta)} + \tfrac{1}{2}H\frac{\delta I}{\delta P},$$

where c has the value $-\cdot 056 \times 10^{-14}$ at the temperature 6° C., and where the second and third terms on the right-hand side are numerically very small in comparison with the first and fourth.

[Added May 15.—The result that the magnetic contraction in nickel, corrected in the manner described above for the effects of Kirchhoff's stresses, is under certain conditions approximately proportional to I^4, is at present to be regarded as purely empirical, and without further experiments it cannot be said to be generally true. It is, therefore, proposed to continue the investigation by repeating the experiments on the nickel wire under different conditions, especially as regards temperature.

It should be borne in mind that in the deduction of the theoretical value of the magnetic contraction, the material is supposed to be perfectly "soft," and no account is taken of hysteresis. Some experiments made by Nagaoka on an ellipsoid of nickel ('Wied. Ann.,' 53, p. 496, 1894) seem to show, however, that the contraction in this metal depends only on the value of the magnetisation, being almost independent of the manner in which that value has been reached. It is, therefore, unlikely that any considerable discrepancy can arise in consequence of hysteresis.]